① Stigler's Diet Problem

Sets
N set of all nutrients
F set of all foods

Parameters

a_{ij} amount of nutrient j per \$ for food i, $\forall j \in N, i \in F$

b_j recommended intake of nutrient j, $\forall j \in N$

Decision Variables

x_i spend on food i $\forall i \in F$, $x_i \in \mathbb{R}$ (since no information is available on the nature of the food items)

Objective function

$$f(x_1 \dots x_n) = x_1 + x_2 + \dots + x_n$$

Constraints

$$\sum_{\substack{i=0 \\ i \in F}} a_{ij} x_i \geq b_j, \forall j \in N$$

$$x_i \geq 0 \quad \forall i \in F$$

The Complete Problem

$$\min_{x_1 \dots x_n} (x_1 + x_2 + \dots + x_n)$$

$$\sum_{\substack{i=0 \\ i \in F}} a_{ij} x_i \geq b_j, \forall j \in N$$

$$x_i \geq 0 \quad \forall i \in F$$

The Solution

a_{ij}

	Food Item	Calories	Protein	Calcium	Iron	Vit A	Thiamine	Riboflavin	Niacin	Ascorbic Acid	Spend	
	1	44700	1411	2	365	0	55.4	33.3	441	0	0.0295	
	2	8400	422	15.1	9	26000	3	23.5	11	60	0	
	3	20600	17	0.6	6	55800	0.2	0	0	0	0	
	4	7400	448	16.4	19	28100	0.8	10.3	4	0	0	
	5	2200	333	0.2	139	169200	6.4	50.8	316	525	0.0019	
	6	2400	138	3.7	80	69000	4.3	5.8	37	862	0	
	7	2600	125	4	36	7200	9	4.5	26	5369	0.0112	
	8	5800	166	3.8	59	16600	4.7	5.9	21	1184	0	
	9	14300	336	1.8	118	6700	29.4	7.1	198	2522	0	
	10	1100	106	0	138	918400	5.7	13.8	33	2755	0.0050	x_i
	11	9600	138	2.7	54	290700	8.4	5.4	83	1912	0	
	12	8500	87	1.7	173	86800	1.2	4.3	55	57	0	
	13	12800	99	2.5	154	85700	3.9	4.3	65	257	0	
	14	17400	1055	3.7	459	5100	26.9	38.2	93	0	0	
	15	26900	1691	11.4	792	0	38.4	24.6	217	0	0.0610	
b_j	Rec Intake	3000	70	0.8	12	5000	1.8	2.7	18	75	0.1087	TOTAL COST
	Act Intake	3000	147.4135	0.8	60.4669	5000	4.1204	2.7	27.3160	75		

② Bets and Arbitrage

Some assumptions: ① Investors maximize their expected returns ② Investors' knowledge of the probability of events is governed by a fixed prior ③ All three sets are played irrespective of the result in the first two and the odds and beliefs are common across sets.

Parameters

b: budget

α: odds of a set win for N

β: odds of a match win for N

p_N: better's belief that N wins a set $(0 \leq p_N \leq 1)$

q_N: better's belief that N wins the match $(0 \leq q_N \leq 1)$

Decision variables

x_{1N}: amount bet on N winning set 1

x_{1J}: — " — J — " —

$x_{2N}, x_{2J}, x_{3N}, x_{3J}$ (as above)

x_{4N}: amount bet on N winning the match

x_{4J}: — " — J — " —

Objective function

$$\mathbb{E}[R] = p_N \left(\frac{x_{1N}}{\alpha} - x_{1J}\right) + (1-p_N)(\alpha x_{1J} - x_{1N}) + \dots$$

prob. of N winning a set

$$\therefore \mathbb{E}[R] = p_N \left(\frac{x_{1N}}{\alpha} - x_{1J}\right) + (1-p_N)(\alpha x_{1J} - x_{1N}) +$$
$$p_N \left(\frac{x_{2N}}{\alpha} - x_{2J}\right) + (1-p_N)(\alpha x_{2J} - x_{2N}) +$$
$$p_N \left(\frac{x_{3N}}{\alpha} - x_{3J}\right) + (1-p_N)(\alpha x_{3J} - x_{3N}) +$$
$$q_N \left(\frac{x_{4N}}{\alpha} - x_{4J}\right) + (1-q_N)(\alpha x_{4J} - x_{4N})$$

Constraints

$$x_{1N} + x_{1J} + \dots + x_{4N} + x_{4J} \leq b$$

$$x_{1N}, x_{1J} \dots \geq 0$$

The problem

$$\max \ \mathbb{E}[R]$$

$$x_{1N} + x_{1J} + \dots \leq b$$

$$x_{1N}, x_{1J} \dots \geq 0$$

this ensures that we do not make a loss (zero is always feasible)

Comments: Investing according to the odds i.e. $p_N = \frac{\alpha}{1+\alpha}$ and $q_N = \frac{\beta}{1+\beta}$, will lead to zero expected return. Hence one needs to "believe better" than the bookmaker in order to make money.