OR Lecture 5

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More about linear programs

Can we reformulate a general mathematical program as a linear program?

$\max \min(3x+2,\ 5x-1)$ — piecewise linear

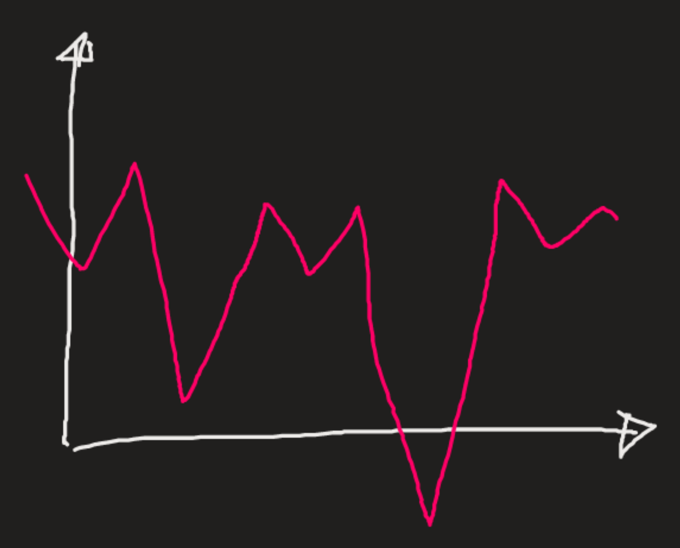
s.t. $x > 0$

$\max\ s$

s.t. $s \leq 3x+2$

$s \leq 5x-1$

$x \geq 0$



how about this piecewise linear objective function

What about $\min |x-y|$ $\equiv$ $\min t$

$x \geq 5, y \geq 2$ $x-y \leq t$ ← linear program

$(x,y) \in \mathbb{R}^2$ $y-x \leq t$

$x \geq 5$

$y \geq 2$

$(x,y,t) \in \mathbb{R}^3$

What about $\min |x_1+y_1| + |y_1-x_2|$

$x_1 \geq 1$

$y_1 \geq 1$

$x_2 \geq 1$

Can be shown that this can be written as a linear program

Thm: A Mathematical programming problem involving a piecewise linear function f can be reformulated as a linear programming problem iff

$\min f(x)$ f is piecewise linear and convex

s.t. $x \in P$

eg $\min\ x+y$

s.t. $\frac{x}{y} \leq t$

$x \geq 0$

$y \geq 1$

$(x,y,t) \in P \subseteq \mathbb{R}^3$

This cannot be reformulated as a linear programming problem.

Solution Procedures

(a) **Heuristics**: Computational shortcuts to easily find a "good" feasible solution.

- no guarantees on the solution qualities
- rule of thumb
- "educated guess"

Recall the knapsack problem, $\max\ u'x \equiv \sum_{i=1}^{n} u_i x_i$

s.t. $c'x = \sum_{i=1}^{n} c_i x_i \leq B$

$x \in \{0,1\}^n$

Arrange $\frac{u_i}{c_i}$ in descending order. Start "filling up" by using as much of quantity i as available ← one heuristic

(b) **Approximation Algorithms** – used typically for NP-hard problems

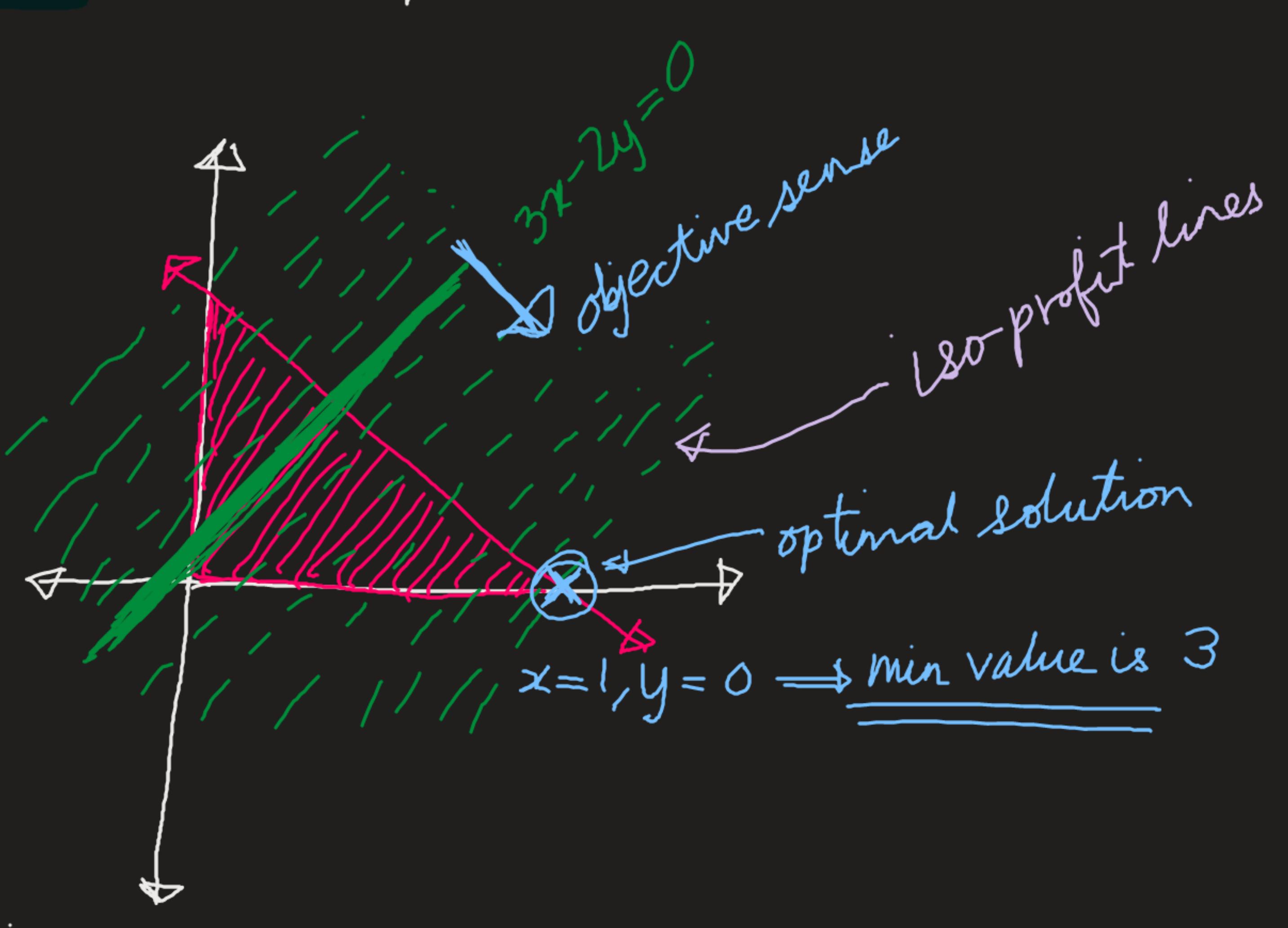
Approximation algorithms come up with "good" polynomial-time solutions to such problems, but with guaranteed accuracy estimates.

(c) **Exact solution methods** – solution procedures to obtain "the" optimal solutions.

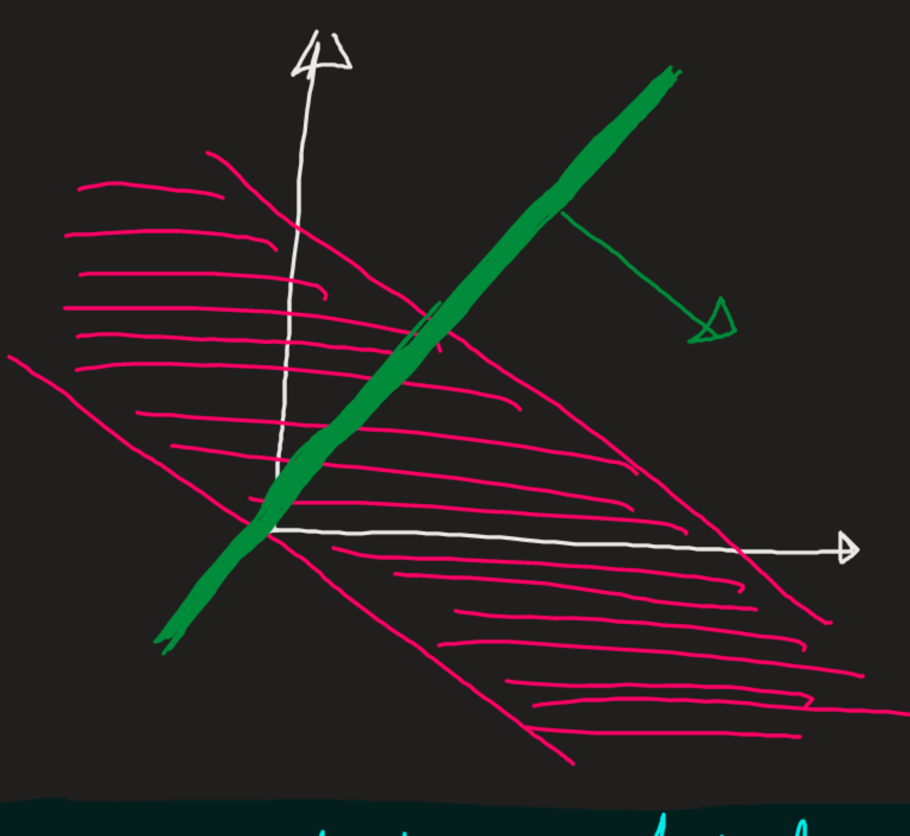
eg $\min\ 3x-2y$

s.t. $0 \leq x+y \leq 1$

$x, y \geq 0$



What if the constraint $x, y \geq 0$ is omitted?



The problem is UNBOUNDED.

I can always move in the direction of the objective sense and obtain a "better" minimum!

Summary of the graphical solution method

(i) Draw the feasible region, polyhedron P. If $P = \emptyset$, then the problem is infeasible.

(ii) Draw the objective contour lines (ISOPROFIT/ISOCOST)

(iii) Move the isoprofit/isocost lines in the optimal direction, until you can't move any more without leaving the polyhedron P.

(iv) If you hit a face/vertex, that is the optimal solution, else the problem is unbounded!